Section 3.3 - Finding Critical Numbers at “Sharp Corners”

Example: Given the function $y = (x^2 - 4)^{2/3}$ (graphed at the right), apply the first derivative test to determine the intervals over which the function is increasing and/or decreasing.

$$y' = \frac{2}{3}(x^2 - 4)^{-2/3}(2x) = \frac{4x}{3(x^2 - 4)^{2/3}}$$

To determine the values where $y' = 0$, set the numerator $= 0$.

Thus, $4x = 0$, so $x = 0$. Yet observe the graph. A relative minimum occurs at $x = -2$ and $x = 2$.

These represent $x$-values where the derivative of the function is “undefined”. Verify this by evaluating $y'(-2)$ and $y'(2)$.

To determine ALL the critical numbers for a given function $f(x)$, you must consider values where $f'(x) = 0$ and also where $f'(x)$ is undefined. To determine where $f'(x)$ is undefined, determine where the denominator $= 0$.

Solve: $3(x^2 - 4)^{2/3} = 0$

<table>
<thead>
<tr>
<th>Interval</th>
<th>$(-\infty,-2)$</th>
<th>$(-2,0)$</th>
<th>$(0,2)$</th>
<th>$(2,\infty)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test value</td>
<td>$-\infty$</td>
<td>$-2$</td>
<td>$0$</td>
<td>$2$</td>
</tr>
<tr>
<td>“sign” of $y'$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inc/dec intervals</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative max/min</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Now complete the test-chart for the critical numbers.

Use this procedure to determine the critical numbers for the following functions, then apply the first derivative test to determine the intervals over which the function is increasing and/or decreasing. Then determine the coordinates of the relative extrema. Use your calculator to graph the function to verify your results.

Examples:
1) $f(x) = x^{2/3} - 4$
2) $f(x) = (x+1)^{1/3}$
3) $f(x) = \frac{x+3}{x^2}$
4) $f(x) = x + \frac{4}{x}$

Homework: Page 186 Problems 9, 29, 33, 35, 37