Calculus One – Section 2.2 – Derivatives of Polynomials

When you apply the formula, \( \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \), you are finding what is known as the derivative of the function \( y = f(x) \). This can be referred to as \( f'(x) \) (read as f prime of x), \( y' \) (read as y prime), \( \frac{dy}{dx} \) (the derivative of f(x) with respect to x), or \( \frac{d}{dx} \) (the derivative of y with respect to x).

Using the “limit” procedure, however, is quite cumbersome. You might find the following properties useful.

**Properties of Derivatives**

A. Derivatives of a constant. \( \frac{d}{dx} (c) = 0 \)  

Determine the derivatives of the following function.

_______1. \( f(x) = 6 \)  
_______2. \( f(x) = 14 \)  
_______3. \( f(x) = -20 \)

B. Derivatives of powers. \( \frac{d}{dx} (x^n) = nx^{n-1} \)  

Find the derivatives of the following functions.

_______4. \( f(x) = x^3 \)  
_______7. \( f(x) = x^4 \)
_______8. \( f(x) = x^{-5} \)  
_______9. \( f(x) = x^{-\frac{1}{2}} \)
_______10. \( f(x) = x^{\frac{2}{3}} \)

C. Derivatives of powers with constants – Constant multiple rule. \( \frac{d}{dx} (3f(x)) = 3 \left( \frac{d}{dx} f(x) \right) \)

_______11. \( y = 3x^3 \)  
_______12. \( y = \frac{1}{2} x^2 \)
_______13. \( y = \frac{1}{4} x \)  
_______14. \( y = -4x^{-6} \)
D. Rewriting to find derivatives. Write the following expressions as the product of a constant and a power of x. Then find the derivative. Note: Write your answers with positive exponents. Fractional powers other than 1/2 may be left as fractional powers. Powers of 1/2 should be written as square roots.

\[ y = \frac{3}{x} \]  

\[ y = \frac{3x^3}{4} \]  

\[ y = 3\sqrt{x} \]  

\[ y = \frac{4}{7\sqrt{x}} \]  

E. Derivatives of polynomials. Sum and/or Difference. Derivatives can be done “term by term”.

\[ y = -3x^2 + 6x \]  

\[ y = 3x + 2 \]  

\[ y = 4x^3 - 6x^2 + 6x + 5 \]  

\[ y = 6 - \frac{4}{x^2} \]  

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F. Expressions can be rewritten to be done term by term.

\[ y = \frac{x^2 - 3x + 4}{x^2} \]  

\[ y = 4x(x^2 + x^3) \]  

\[ y = \frac{4}{\sqrt{x}} + \frac{5}{3\sqrt{x}} \]  

\[ y = 4x^6 + \sqrt{x} - \frac{3}{\sqrt{x}} \]  

\[ y = x(x^3 - x^4 - 3) \]  

\[ y = \frac{3x^4 - 2x^3 + x^2}{x^2} \]  

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