A. Definitions
An expression of the form \( \sqrt[n]{r} \) is a **radical expression** where \( r \) is the radicand and \( n \) is the index. Name the index and the radicand in the following:

\[
\sqrt[3]{a} \quad \sqrt[3]{abc} \quad \sqrt{x + y} \quad \sqrt[3]{5x^2}
\]

**Radical expressions can be simplified using the properties:**

\[
\sqrt[n]{a^n} = |a| \text{ if } n \text{ is even} \quad \sqrt[n]{a^n} = a \text{ if } n \text{ is odd}
\]

Examples:

\[
\sqrt{a^2} = \boxed{a} \quad \sqrt{a^3} = \boxed{a} \quad \sqrt{a^4} = \boxed{a^2} \quad \sqrt{a^5} = \boxed{a^2} \quad \sqrt{a^9} = \boxed{a^3} \quad \sqrt{a^{10}} = \boxed{a^5}
\]

\[
\sqrt{25} = \sqrt{5^2} = |5| = 5 \quad \sqrt{196} = \sqrt{14^2} = |14| = 14
\]

\[
\sqrt{-36} = \sqrt{6^2} = |6| = 6
\]

\[
\sqrt[4]{125} = \sqrt[4]{5^3} = \boxed{5} \quad \sqrt[3]{-27} = \sqrt[3]{(-3)^3} = -3
\]

Notes:
- **When the radicand is numeric** the absolute value step is not required.
- **When the radicand is a variable** the absolute value step is often omitted and the variable is assumed to represent a positive value.
- **Remember, when the index is even** a negative radicand yields an undefined value.
- **When the index is odd** a negative radicand can be evaluated

Review examples - Simplify the following.

1. \( \sqrt[5]{3^{15}} \)  
2. \( \sqrt[3]{p^8} \)  
3. \( \sqrt[3]{27} \)  
4. \( \sqrt[4]{256} \)  
5. \( \sqrt[5]{1} \)  
6. \( \sqrt{-64} \)

7. \( \sqrt{-64} \)  
8. \( \sqrt[3]{1024} \)  
9. \( \sqrt[3]{81} \)  
10. \( -\sqrt{144} \)  
11. \( \sqrt[3]{19^7} \)
B. More complex radical expressions can be simplified by factoring the radicand and applying the properties described above.

Examples:

$$\sqrt[3]{a^6} = \sqrt[3]{(a^3)^2} = |a^3| = |a|^3$$ (often omitting absolute value)  
$$\sqrt[3]{a^{12}} = \sqrt[3]{(a^4)^3} = a^4$$

Use these properties to simplify the following expressions

1. $\sqrt[3]{a^{12}}$  
2. $\sqrt[3]{a^8}$  
3. $\sqrt[3]{a^{20}}$  
4. $\sqrt[3]{a^{21}}$  
5. $\sqrt[3]{a^{64}}$

****Can you identify a "shortcut" relating the index and the power to use in simplifying radical expressions with variable radicands?

C. To simplify radical expressions involving factors, use the property of multiplication with radicals:  

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$  (Note: this property works for any index.)

Examples:

$$\sqrt{a^3 b^2} = \sqrt{a^3} \cdot \sqrt{b^2} = a \cdot b = ab$$  
$$\sqrt[3]{64 a^{12}} = \sqrt[3]{64} \cdot \sqrt[3]{a^{12}} = _____$$

Use this property to simplify the following expressions.

1. $\sqrt{a^8 b^3}$  
2. $\sqrt[3]{a^6 b^{12}}$  
3. $\sqrt[3]{25 a^2}$  
4. $\sqrt[3]{36 a^2 b^6}$  
5. $\sqrt[3]{-125 a^6}$

D. To simplify radical expressions involving quotients, use the property of division with radicals:

$$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$  (Note: This property works for any index)

Use this property to simplify the following expressions.

1. $\sqrt[4]{\frac{49}{16}}$  
2. $\sqrt[6]{\frac{a^{10}}{b^{12}}}$  
3. $\sqrt[3]{\frac{27 a^9}{b^{15}}}$  
4. $\sqrt[5]{\frac{1}{243}}$

You Try:

1. $\sqrt[3]{81 a^{10}}$  
2. $\sqrt[3]{216 y^{15}}$  
3. $\sqrt{32 x^{35}}$  
4. $\sqrt[3]{\frac{a^{18}}{b^{27}}}$

Hw: Page 603 Problems 31, 32, 41-48, 55-60, 69, 71, 75