Calculus Test—Chapter 8

Change the following.

1. Cartesian \((5\sqrt{3}, -5)\) to polar coordinates
\[
\left(\frac{-5\sqrt{3}}{2}, \frac{5\pi}{4}\right)
\]
2. Polar \((5, \frac{5\pi}{4})\) to rectangular (cartesian) coordinates
\[
\left(-\frac{5\sqrt{2}}{2}, -\frac{5\sqrt{2}}{2}\right)
\]

Change the following equations.

3. Polar \(r = 3\sin \theta \csc \theta\) to Cartesian equations
4. Cartesian \(x^2 + 3y^2 = 4x\) to polar equations

Identify the following polar equations.

5. r = \(3 - 4\cos \theta\)
6. r = \(4\cos 3\theta\)
7. r = \(3 - 4\sin 2\theta\)
8. r = 8
9. r = \(5\cos \theta\)
10. r = \(4 - 4\sin \theta\)
11. \(\theta = \frac{\pi}{4}\)

12. Graph the equation \(r = 3\theta\)
13. Graph the equation $r = 3 \sin \theta$.

| $\theta$ |
|---|---|
| 0   | 0 |
| $\frac{\pi}{2}$ | 3 |
| 0   | $\frac{3\pi}{2}$ |
| -3  | $\frac{\pi}{2}$ |
| 0   | $2\pi$ |

14. Graph the equation $r = 4 \sin 3\theta$.

| $\theta$ |
|---|---|
| 0   | 0 |
| $\frac{\pi}{4}$ | 4 |
| 0   | $\frac{\pi}{3}$ |
| -4  | $\frac{2\pi}{3}$ |
| 0   | $2\pi$ |

15. Find the area of the region bounded by $r^2 = 4 \cos 2\theta$.

\[
A = \frac{1}{2} \int_{0}^{\pi/4} 4 \cos 2\theta \, d\theta = 2 \int_{0}^{\pi/4} \cos 2\theta \, d\theta = \sin \frac{\pi}{2} - \sin 0 = 1, \quad 4 \text{ pieces} = 4
\]

16. Find the area of the region bounded by $r = 4 \cos \theta$.

\[
A = \frac{1}{2} \int_{0}^{\pi/2} (4 \cos \theta)^2 \, d\theta = 16 \int_{0}^{\pi/2} \cos^2 \theta \, d\theta = 8 \int_{0}^{\pi/2} \frac{1 + \cos 2\theta}{2} \, d\theta
\]

\[
A = 4 \int_{0}^{\pi/2} (1 + \cos 2\theta) \, d\theta = 4 \left[ \theta + \frac{1}{2} \sin 2\theta \right]_{0}^{\pi/2} = 4 \left( \frac{\pi}{2} + 0 \right) - \left( 0 + 0 \right) = 4 \pi
\]

* (maybe I've done and doubled)