Differential Equations Test
Word Problems

Name______________________

Solve the following.

1. A boy goes sledding down a long 30 degree slope. The combined weight of the boy and the sled is 72 pounds. The air resistance is numerically equal to twice their velocity. If they started from rest and their coefficient of friction is .5, what is their velocity after 1 sec?

2. Suppose that an 80-microgram sample of radioactive isotope decays to 70 micrograms after 10 hours. What is the half-life of the isotope (or how long does it take for only half of the original sample to remain?)
3. A circuit has in series an electromotive force of 5 sin 100t V, a resistor of 10 ohms, an inductor of 0.25 henries and a capacitor of 4 \times 10^{-4} farads. If the initial current and the initial charge on the capacitor is both zero, find the charge on the capacitor at any time t.

4. A spring is such that a 2-lb weight stretches it ½ ft. An impressed force \( \frac{1}{4} \sin 8t \) and a damping force of magnitude \( \frac{dx}{dt} \) are both acting on the spring. The weight starts \( \frac{1}{4} \) ft below the equilibrium point with an imparted upward velocity of 3 ft/sec. Find a formula for the position of the weight at time t.
5. A ball weighing 6 lb is thrown vertically downward toward the earth from a height of 1000 ft with an initial velocity of 6 ft/sec. As it falls it is acted upon by air resistance that numerically equal to $\frac{2}{3}v$ (in pounds) where $v$ is the velocity.

6. An object weighing 32 pounds is released from rest 50 ft above the surface of a calm lake. Before the object reaches the surface of the lake, the air resistance is given by $2v$, where $v$ is the velocity. After the object passes beneath the surface, the water resistance is given by $6v$. Further, the object is then buoyed up by a buoyancy force of 8 pounds, find the velocity of the object 2 sec after it passes beneath the surface of the lake. (Please show all your work.)